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LOCAL ISOTROPY AND REFRACTIVE INDEX FLUCTUATIONS IN THE SURFACE LAYER OF THE ATMOSPHERE

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ABSTRACT

In the air layer near the ground, atmospheric probing by optical means is influenced by turbulent fluctuations of refractive index. Spectral characteristics of the fluctuations may be significant, for example, in determining mean vertical temperature variations by measuring mean refraction of a horizontal beam. Theoretical and experimental evidence for the existence of local isotropy is briefly examined and conflicting results are found. Recent measurements of temperature spectra support earlier hot wire anemometer and optical scintillation measurements that show little evidence of local isotropy at 1 to 1.5 meters over an extensive uniform and level grass covered field.

1. INTRODUCTION

Characteristics of turbulent fluctuations of refractive index in an optical path usually cannot be ignored to understand or to predict atmospheric effects on optical propagation. Neither can they be ignored if one wishes to deduce atmospheric information from the effects. In the preceding paper, Lawrence gives examples to illustrate the latter point. This note is intended to raise a question about our knowledge of the spectral characteristics of refractive index fluctuations in the layer of air near the ground, a part of the atmosphere often used to study how turbulence influences optical propagation. The specific point to be examined is whether or not there exists a significant size range of turbulent motions that are isotropic at heights of 1 to 2 meters over an extended horizontal plane of uniform roughness. For isotropic turbulence, refractive index spectra may be expected to be proportional to the frequency, or wave number, raised to the $-5/3$ power in accordance with the theories of Kolmogorov (1941) and Obukhov (1949). For horizontally homogeneous turbulence with thermal stratification it is reasonable to expect wind and temperature spectra to depend on buoyancy forces. This is the case of interest here because only when there is a mean vertical temperature

ISOTROPY AND REFRACTIVE INDEX FLUCTUATIONS

gradient will there be significant index of refraction fluctuations. Monin (1962) considered this influence and concluded that temperature spectra, for both lapse and inversion conditions, decrease with frequency at a rate less than that for isotropic turbulence.

2. TURBULENT REFRACTIVE INDEX FLUCTUATIONS AND MEAN REFRACTION MEASUREMENTS

The significance of isotropy for remote probing of the atmosphere may be illustrated with the example given by Lawrence (loc. cit.) of determining the space average temperature variation with height by measuring the mean refraction of an optical beam. Fleagle (1950) derived a relationship to show that the apparent difference in height between a distant object and its image is proportional to the square of the viewing distance for a given average temperature difference. Practical use of the relationship is hindered, however, by the fact that turbulent index of refraction fluctuations cause the image to fluctuate in size, brightness and position making it difficult, if not impossible, to measure accurately the image position. Tatarski (1961) shows that the variance of the logarithmic intensity fluctuations is proportional to the $11/6$ power of the path length if the small scale turbulence is isotropic and the spectrum of index of refraction fluctuations follows the $-5/3$ law in accordance with Kolmogorov's inertial subrange. The result is that the turbulence imposed intensity fluctuations increase with path length nearly as fast as the apparent height difference itself so that an increase in path length may not in fact produce a measurement advantage indicated by Fleagle's equation.

If, on the other hand, the turbulence is not isotropic fluctuation effects may vary with the path length raised to a power different from $11/6$ and advantages or disadvantages will be experienced accordingly. Portman et al., (1962) reported experimental results for path lengths between 122 and 610 meters (1.5 meters high) in which the power of the path length was found to correspond to 1.56, 1.76 and 1.80 for Richardson numbers of +0.03, +0.011 and -0.065, respectively. The findings indicate a small advantage to be gained by increasing path length to increase precision in measuring optical beam refraction.

For longer paths and stronger turbulence these effects may not be significant since experimental results given by Siedentopf and Wisshak (1948), Gracheva and Gurvich (1965) and Gracheva (1967) show that the path length influence is absent for paths greater than about 1 km. Thus, increasing the optical path beyond 1 km should give advantage in measuring thermal stratification optically. The theories of Tatarski (1967) and of deWolf (1968) show that the critical length depends on the power of the path length. Both authors assume it to be $11/6$ (again, from the $-5/3$ law) but if the actual power is less than $11/6$, the critical length is less and vice versa. In all cases the critical length depends on the strength of turbulent fluctuations as well as the wave length of light.

3. THEORETICAL BASIS FOR ISOTROPY

The question of the existence of the Kolmogorov inertial subrange within a few meters of the ground has been examined both theoretically and experimentally. Most results seem to establish its reality at heights of a meter and above in spite of the fact that Kolmogorov's basic conditions for its existence apparently are absent. This is to say, neither wind shear in the mean motion

nor buoyant energy, the two sources of turbulent energy, satisfy the Kolmogorov requirement that turbulent energy feeding be characterized by scales much larger than those characterizing the smallest motions. It is difficult not to agree with Corrsin (1958) and Monin (1965) in their suggestions that both shear and buoyancy must work at all scales of motion in the layer of air near the ground.

Corrsin's analysis is for shear flow in the absence of buoyancy forces. By considering the characteristic times required for turbulent energy to cascade to smaller scales and to transfer to other components of motion, he concludes that the low wave number limit, κ_l , and upper wave number limit, κ_u , of the inertial subrange are given by the inequalities

$$\kappa_l \gg \epsilon^{-1/2} \left(\frac{1}{2} \frac{d\bar{u}}{dz} \right)^{3/2}, \quad \kappa_u \ll \left(\frac{\epsilon}{\nu^3} \right)^{1/4}$$

These reduce to

$$\kappa_l \gg (kZ)^{-1} \left(\frac{1}{2} \right)^{3/2}, \quad \kappa_u \ll \left(\frac{u_*^3}{\nu^3 kZ} \right)^{1/4}$$

for the usual relationships

$$\epsilon = \frac{u_*^3}{kZ} \quad \text{and} \quad \frac{d\bar{u}}{dz} = \frac{u_*}{kZ}$$

for the constant flux layer next to the ground. If the height $z = 100$ cm, von Karman number $k = 0.4$, and kinematic viscosity $\nu = 0.15 \text{ cm}^2 \text{ sec}^{-1}$, and for a friction velocity $u_* = 45 \text{ cm sec}^{-1}$, corresponding to a two meter wind speed $u = 500 \text{ cm sec}^{-1}$, the conditions are, approximately,

$$\kappa_l \gg 0.009 \quad \text{and} \quad \kappa_u \ll 28$$

It appears, therefore, that an inertial subrange of limited extent, say $0.1 < \kappa < 2.8$, is to be expected in neutral conditions at 1 meter above ground if the double inequality signs can be interpreted as representing an order of magnitude.

Lumley and Panofsky (1964) extend the analysis to include buoyancy effects. With some assumptions (generally valid only in near neutral conditions) they conclude that in stable stratification the existence of the inertial subrange can, on the average, be assumed for heights above 1 meter. For unstable stratification their estimate is that

$$\text{or} \quad z \geq 100 \nu / u_*$$

$$z \geq 1/3 \text{ cm for } u_* = 45 \text{ cm sec}^{-1} \text{ and } \nu = 0.15 \text{ cm}^2 \text{ sec}^{-1}$$

if the inertial subrange is to exist.

4. EXPERIMENTAL FINDINGS

Experimental information on the existence of the inertial subrange may be found in several forms. The most commonly cited are spectral relationships for wind component and temperature fluctuations. In general one dimensional spectral densities are found to be proportional to the $-5/3$ power of frequency or wave number. (See, for example, Gurvich, 1962; and Tsvang, 1960.) Other conditions used to test the hypothesis are the absence of Reynolds fluxes of heat and momentum for the subrange and the fact that the spectral densities of the cross wind (transverse) components are $4/3$ times that of the along wind (longitudinal) component in accordance with the analysis of von Karman and Howarth (1938) for isotropic turbulence.

MacCreedy (1962) examined data from a number of sources to describe the inertial subrange within 200 meters of the ground. His summary indicates that the inertial subrange at one meter extends between eddies on the order of 0.1 and 150 cm. He found, also, that one-dimensional velocity spectra characteristically are proportional to $-5/3$ power of wave number at wave numbers lower than those at which isotropy can be said to exist, a fact that is significant if one attempts to identify the inertial subrange from one-dimensional velocity spectra alone.

In a recent study of turbulence spectra, Busch and Panofsky (1968) examined Round Hill data (heights of 15, 16, 40, 46 and 91 meters) in regard to the Karman-Howarth $4/3$ relationship. They concluded that: "in regions over which the spectra obey $-5/3$ power laws, the ratio of the lateral to the longitudinal spectra shows fair agreement with the $4/3$ ratio predicted by the Kolmogorov hypothesis for the inertial subrange. The vertical-longitudinal ratio has a similar tendency." Examination of their diagrams, however, shows that almost all data for the lower values of z/λ_N , in which λ_N is the Nyquist spatial frequency, i.e., the highest frequency possible to measure for each level, all fall well below the $4/3$ value. In fact, for the vertical-to-longitudinal spectral density ratios, 50 percent of the data have values equal to or less than unity. For the transverse-to-longitudinal spectral density ratios, more than 50 percent of the data have values less than $4/3$. It is difficult to see how the authors arrived at their conclusion on this criterion for the inertial subrange.

Hot wire anemometer measurements were made by Biggs (1966) at a meter above a horizontal and uniformly cut grass surface at Willow Run airfield. (See Portman, Ryznar and Waqif, 1968.) Spectra for u, v, and w were obtained for five 12-minute periods. Richardson numbers for four of the periods were all about -0.01 and for the fifth about +0.07. All spectra showed an approximate $-5/3$ power wave number dependency between wave numbers of 0.01 and 1.0 rad/cm but in all five cases the longitudinal (u) spectral densities were greater than comparable spectral densities for the transverse components.

Temperature spectra were not measured, unfortunately, for the above periods of hot wire measurements. More recent measurements, however, have been made at the same location and are soon to be reported. Preliminary analysis for averaged six minute temperature spectra yield the following results:

<u>No. of spectra averaged</u>	<u>Richardson No.</u>	<u>Slope on a ln power-ln wave no. plot</u>
6	-0.025	-1.25
5	+0.015	-1.33
7	-0.027	-1.11, -1.50*
5	-0.062	-1.13, -1.50*

* The first number applies for wave numbers less than 2.5×10^{-2} rad/cm and the second for wave numbers greater than 2.5×10^{-2} rad/cm.

Hot wire anemometer measurements to accompany these data have not yet been processed. These results deviate significantly from the $-5/3$ power law and support the earlier hot wire anemometer measurements, made at the same location under similar conditions, in showing no evidence of isotropy.

Brightness fluctuations within a laser beam (scintillation) were measured along with temperature and wind component spectra at the Willow Run field station. (The optical path was 500 meters long and about 1.5 meters above ground.) Scintillation spectra are related to refractive index spectra (Tatarski, 1961) and it might be expected that deviations from isotropy would appear in the measured scintillation spectra. For one set of measurements (Portman, Ryznar and Waqif, 1968) it was found that spectra for inversion conditions showed a marked suppression of low frequency scintillation and those for lapse conditions showed a weak low frequency enhancement, both in comparison to Tatarski's isotropic model. Subsequent measurements, however, have been partially inconsistent with this finding. It appears that representation of the refractive index spectra by a simple power law may be inadequate to relate temperature spectra, such as those given above, to scintillation spectra. If such is the case, it is evident that the spectral regions significant for optical diffractive effects are not isotropic for these conditions.

5. CONCLUSIONS

Spectral characteristics of index of refraction fluctuations in the layer of air near the ground are significant for understanding atmospheric influences on optical propagation, and therefore, for probing the atmosphere with optical techniques. Both theoretical and experimental findings are inconclusive in regard to whether or not an inertial subrange of significant wave number interval actually exists at heights of 1 to 2 meters above ground. Most experimental evidence is based on the observation of a $-5/3$ power dependency on wave number, a relationship that is known to exist in anisotropic regions of the spectrum. Relationships between wind component spectral densities, careful examination of temperature spectra, and certain optical scintillation measurements, however, do not support the existence of the inertial subrange at this height. Since wave number limits of the inertial subrange are expected to increase in separation with increasing height, optical experiments that are strongly dependent on the existence of isotropy and a $-5/3$ power law should be conducted at heights well above the ground. There is an important need for more information on the structure of turbulence near the ground and how the spectral characteristics of refractive index fluctuations depend on both the structure and the height above ground.

ISOTROPY AND REFRACTIVE INDEX FLUCTUATIONS

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